Regular expressions

Consider the regular sets denotated by the following pairs of regular expressions, with $\Sigma = \{a, b, c\}$. For each pair, say whether the two corresponding languages are equal. If not, give an example of a word in one that is not in the other

- $(a + b)^*$ and $a^* + b^*$
- $a(bca)^*bc$ and $ab(cab)^*c$
- $\emptyset^*$ and ϵ^*
- $(a^*b^*)^*$ and $(a^*b)^*$
- $(ab + a)^*a$ and $a(ba + a)^*$

Pumping Lemma

If $w \in \{0, 1\}^*$ we write $\#i(w)$ the number of occurences of i in w (with $i =$ 0 or 1). Show that the following language is not regular

$$L = \{w \in \{0, 1\}^* \mid \#0(w) = 2 \times \#1(w)\}$$

and similarly, that the following language is not regular

$$M = \{w \in \{0, 1\}^* \mid \#0(w) \leq \#1(w) \leq \#0(w) + 1\}$$

(hint: look at example 4.2). Show however the following language is regular

$$N = \{w \in \{0, 1\}^* \mid \#0(w) \times \#1(w) \text{ is even}\}$$

Equivalence relations

We recall that a partition σ of a set X is a set of nonempty subsets $A \subseteq X$ such that

- For any $x \in X$ there exists $A \epsilon \sigma$ such that $x \epsilon A$,
- If $A, B \epsilon \sigma$ and $A \cap B \neq \emptyset$ then $A = B$.

An element of σ is also called a *block* or *cell* of the partition σ.

To any partition σ of a set X we associate the equivalence relation $R(\sigma)$ defined by

$$R(\sigma)\ x\ y \equiv (\exists A \epsilon \sigma)[x \epsilon A \wedge y \epsilon A].$$

Exercice 1: We say that σ and τ are *independent* iff for any A block of σ and B block of τ we have $A \cap B \neq \emptyset$. Show that if τ and σ are independent then for any x, y there exists z such that $R(\tau)\ x\ z$ and $R(\sigma)\ z\ y$.

Exercice 2: If R and S are equivalence relations show that $R \cap S$ is an equivalence relation. If σ and τ are two partition, we define δ to be the set of $C \subseteq X$ such that there exists $A \epsilon \sigma$ and $B \epsilon \tau$ such that $A \cap B \neq \emptyset$ and $C = A \cap B$. Show that δ is a partition and that $R(\delta) = R(\sigma) \cap R(\tau)$.

Minimal automata

Minimize the following automata

	a	b
$\rightarrow$1	6	3
2	5	6
$*$3	4	5
$*$4	3	2
5	2	1
6	1	4

	a	b
$\rightarrow$1	2	3
2	5	6
$*$3	1	4
$*$4	6	3
5	2	1
6	5	4