Kurs: MAN321/TMV026 Ändliga automater och formella språk
Plats: M-huset
Tid: 08.30-12.30
Datum: 2007-08-27 No help documents
Telefonvakt: Thierry Coquand, 7721030
The questions can be answered in english or in swedish.
total 30; ≥13: 3, ≥19: 4, ≥25: 5
total 30; ≥13: G, ≥21: VG

1. Let Σ be an alphabet. What is, mathematically, a deterministic finite automaton on Σ (1p)? Explain what is the language determined by such a finite automaton (1p). Explain why such a language is a context-free language (1p).

2. Minimize the following automaton (2p)

	a	b
→0	1	3
1	0	3
2	1	4
∗3	5	5
4	3	3
∗5	5	5

3. Build a NFA with 3 states that accepts the language $\{ab, abc\}^*$. (2p)

4. Build a DFA corresponding to the regular expression $(ab)^* + a^*$. (3p)

5. Let Σ be $\{0, 1\}$. We recall that the regular expressions are on Σ are given by the grammar

$$E \ ::= \ 0 \mid 1 \mid \emptyset \mid \epsilon \mid E + E \mid EE \mid E^*$$

Give a regular expression E such that

$$L(E) = \Sigma^* - L(10(0 + 1)^*) \quad (2p)$$

6. Build a DFA that recognizes exactly the word in $\{0, 1\}^*$ ending with the string 1110. (2p)

7. Is the following grammar ambiguous? Why (2p)?

$$S \to AB \mid aaB, \qquad A \ \to \ a \mid Aa, \qquad B \to b$$

Construct an unambiguous grammar which is equivalent to this grammar (2p).

8. Consider the grammar

$$S \;\rightarrow\; aaB, \qquad A \;\rightarrow\; bBb \mid \epsilon, \qquad B \;\rightarrow\; Aa$$

Show that the string *aabbabba* is *not* in the language generated by this grammar (3p).

9. Find context-free grammars for the languages

   (a) $L = \{\ a^n b^n c^k \mid n \leq k\}$ (1p)

   (b) $L = \{\ a^n b^m \mid n \neq m\ \}$ (1p)

10. Let L, M, N be languages on an alphabet Σ^* (that is, we have L, M, N subsets of Σ^*). Explain why we have $L(M \cap N) \subseteq LM \cap LN$ (2p). Give an example showing that we do not have $LM \cap LN \subseteq L(M \cap N)$ in general (2p).

11. Let Σ be an alphabet and let L_1, L_2 be subsets of Σ^*. Assume that $L_1 \cap L_2 = \emptyset$ and L_1 is finite and that $L_1 \cup L_2$ is regular. Can we deduce that L_2 is regular (3p)?